Connecting Supertrees (supertrees)

Gardens by the Bay is a large nature park in Singapore. In the park there are n towers, known as supertrees. These towers are labelled 0 to $n-1$. We would like to construct a set of **zero or more** bridges. Each bridge connects a pair of distinct towers and may be traversed in **either** direction. No two bridges should connect the same pair of towers.

A path from tower x to tower y is a sequence of one or more towers such that:

- the first element of the sequence is x,
- the last element of the sequence is y,
- all elements of the sequence are **distinct**, and
- each two consecutive elements (towers) in the sequence are connected by a bridge.

Note that by definition there is exactly one path from a tower to itself and the number of different paths from tower i to tower j is the same as the number of different paths from tower j to tower i.

The lead architect in charge of the design wishes for the bridges to be built such that for all $0 \leq i, j \leq n-1$ there are exactly $p[i][j]$ different paths from tower i to tower j, where $0 \leq p[i][j] \leq 3$.

Construct a set of bridges that satisfy the architect's requirements, or determine that it is impossible.

Implementation details

You should implement the following procedure:

```
int construct(int[][] p)
```

- p: an $n \times n$ array representing the architect's requirements.
- If a construction is possible, this procedure should make exactly one call to `build` (see below) to report the construction, following which it should return 1.
- Otherwise, the procedure should return 0 without making any calls to `build`.
- This procedure is called exactly once.

The procedure `build` is defined as follows:

```
void build(int[][] b)
```

- b: an $n \times n$ array, with $b[i][j] = 1$ if there is a bridge connecting tower i and tower j, or

$b[i][j] = 0$ otherwise.
- Note that the array must satisfy $b[i][j] = b[j][i]$ for all $0 \leq i, j \leq n-1$ and $b[i][i] = 0$ for all $0 \leq i \leq n-1$.

Examples

Example 1

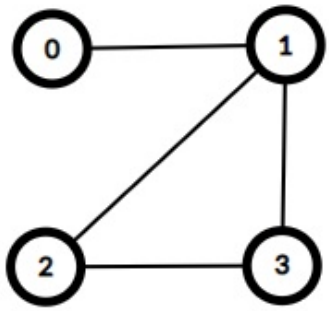
Consider the following call:

```
construct([[1, 1, 2, 2], [1, 1, 2, 2], [2, 2, 1, 2], [2, 2, 2, 1]])
```

This means that there should be exactly one path from tower 0 to tower 1. For all other pairs of towers (x, y), such that $0 \leq x < y \leq 3$, there should be exactly two paths from tower x to tower y. This can be achieved with 4 bridges, connecting pairs of towers $(0, 1)$, $(1, 2)$, $(1, 3)$ and $(2, 3)$.

To report this solution, the `construct` procedure should make the following call:

- `build([[0, 1, 0, 0], [1, 0, 1, 1], [0, 1, 0, 1], [0, 1, 1, 0]])`



It should then return 1.

In this case, there are multiple constructions that fit the requirements, all of which would be considered correct.

Example 2

Consider the following call:

```
construct([[1, 0], [0, 1]])
```

This means that there should be no way to travel between the two towers. This can only be satisfied by having no bridges.

Therefore, the `construct` procedure should make the following call:

- `build([[0, 0], [0, 0]])`

After which, the `construct` procedure should return 1.

Example 3

Consider the following call:

```
construct([[1, 3], [3, 1]])
```

This means that there should be exactly 3 paths from tower 0 to tower 1. This set of requirements cannot be satisfied. As such, the `construct` procedure should return 0 without making any call to `build`.

Constraints

- $1 \leq n \leq 1000$
- $p[i][i] = 1$ (for all $0 \leq i \leq n-1$)
- $p[i][j] = p[j][i]$ (for all $0 \leq i, j \leq n-1$)
- $0 \leq p[i][j] \leq 3$ (for all $0 \leq i, j \leq n-1$)

Subtasks

1. (11 points) $p[i][j] = 1$ (for all $0 \leq i, j \leq n-1$)
2. (10 points) $p[i][j] = 0$ or 1 (for all $0 \leq i, j \leq n-1$)
3. (19 points) $p[i][j] = 0$ or 2 (for all $i \neq j$, $0 \leq i, j \leq n-1$)
4. (35 points) $0 \leq p[i][j] \leq 2$ (for all $0 \leq i, j \leq n-1$) and there is at least one construction satisfying the requirements.
5. (21 points) $0 \leq p[i][j] \leq 2$ (for all $0 \leq i, j \leq n-1$)
6. (4 points) No additional constraints.

Sample grader

The sample grader reads the input in the following format:

- line 1: n
- line $2+i$ ($0 \leq i \leq n-1$): $p[i][0]$ $p[i][1]$ $\ldots$ $p[i][n-1]$

The output of the sample grader is in the following format:

- line 1: the return value of `construct`.

If the return value of `construct` is 1, the sample grader additionally prints:

- line $2+i$ ($0 \leq i \leq n-1$): $b[i][0]$ $b[i][1]$ $\ldots$ $b[i][n-1]$